

## APPENDIX – Common Expressions

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The number of combinations of  $n$  distinct objects taken  $r$  at a time is (from ref. [9], pg. 23):

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}, \text{ where } x! = x * (x-1) * (x-2) * \dots * (x-(x-1)) \quad (\text{A1})$$

If A and B are any two events (ref. [9], pg. 28):

$$\text{Probability}\{A \text{ OR } B\} = \text{Probability}\{A\} + \text{Probability}\{B\} - \text{Probability}\{A \text{ AND } B\} \quad (\text{A2})$$


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### BERNOULLI TRIALS:

A Bernoulli trial can result in success with probability  $p$  and a failure with probability  $q = 1-p$ . The probability distribution of the random variable,  $X$ , representing the number of successes in  $n$  independent trials is given by the Binomial Distribution (from ref. [9], pg. 117):

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n \quad (\text{A3})$$

The probability of *at least*  $r$  successes in  $n$  independent trials is (from ref. [9], pg. 118):

$$1 - P(\# \text{ successes} < r) = 1 - \sum_{x=0}^{r-1} b(x; n, p) \quad (\text{A4})$$


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### SUMMATION EXPRESSIONS:

$$(1-a) \sum_{n=0}^{\infty} a^n = \sum_{n=0}^{\infty} a^n - \sum_{n=0}^{\infty} a^{n+1} = (1 + a^1 + \dots + a^{\infty-1} + a^{\infty}) - (a^1 + a^2 + \dots + a^{\infty} + a^{\infty+1})$$

$$(1-a) \sum_{n=0}^{\infty} a^n = 1 - a^{\infty+1} = 1; |a| < 1$$

$$\boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}; |a| < 1} \quad (\text{A5})$$

$$(1-a) \sum_{n=0}^{N-1} a^n = (1 + a^1 + \dots + a^{N-2} + a^{N-1}) - (a^1 + a^2 + \dots + a^{N-1} + a^N)$$

$$(1-a) \sum_{n=0}^{N-1} a^n = (1 - a^N)$$

$$\boxed{\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}} \quad (\text{A6})$$

## PERIODIC AVERAGES OF A COMPLEX SINUSOID:

$$\begin{aligned}
& \frac{1}{T} \int_0^T e^{\frac{j2\pi nt}{T}} dt; \quad 0 \leq t \leq T \\
& = \frac{T}{jT 2\pi n} e^{\frac{j2\pi nt}{T}} \bigg|_0^T = \frac{T}{jT 2\pi n} - \frac{T}{jT 2\pi n} = 0, \quad n \neq 0 \\
& = \frac{1}{T} \int_0^T e^{\frac{j2\pi 0}{T}} dt = \frac{1}{T} \int_0^T 1 dt = 1; \quad n=0, T, 2T, \text{etc.}
\end{aligned}$$

$$\frac{1}{T} \int_0^T e^{\frac{j2\pi nt}{T}} dt = \delta(n \bmod T)$$

(A7)

$$\begin{aligned}
\sum_{k=0}^{N-1} e^{\frac{j2\pi kn}{N}} &= \frac{1 - e^{\frac{j2\pi nN}{N}}}{1 - e^{\frac{j2\pi n}{N}}} = 0; \quad n \neq 0 \\
&= \sum_{k=0}^{N-1} 1 = N; \quad n = 0, N, 2N, \text{etc.}
\end{aligned}$$

$$\frac{1}{N} \sum_{k=0}^{N-1} e^{\frac{j2\pi kn}{N}} = \delta(n \bmod N)$$

(A8)

## AVERAGE POWER OF CONTINUOUS AND DISCRETE COMPLEX SINUSOIDS:

$$\begin{aligned}
& \frac{1}{T} \int_0^T (Ae^{jk\omega_0 t}) (Ae^{jk\omega_0 t})^* dt = \frac{1}{N} \sum_{n=0}^{N-1} (Ae^{jk\omega_0 n/N}) (Ae^{jk\omega_0 n/N})^* \\
& \frac{1}{T} \int_0^T (Ae^{jk\omega_0 t}) (Ae^{-jk\omega_0 t}) dt = \frac{1}{N} \sum_{n=0}^{N-1} (Ae^{jk\omega_0 n/N}) (Ae^{-jk\omega_0 n/N}) \\
& \frac{1}{T} \int_0^T (A^2 e^{jk\omega_0 t} e^{-jk\omega_0 t}) dt = \frac{1}{N} \sum_{n=0}^{N-1} (A^2 e^{jk\omega_0 n/N} e^{-jk\omega_0 n/N}) \\
& \frac{1}{T} \int_0^T A^2 dt = \frac{1}{N} \sum_{n=0}^{N-1} A^2 \\
& \frac{A^2 T}{T} = \frac{A^2 N}{N} = A^2
\end{aligned}$$
(A9)

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